Optimization of a Constrained Quadratic Function

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Dedicated on the occasion of his 60th birthday to Neculai Andrei in appreciation of a lifetime of contributions to mathematics, as a productive researcher, an author of valuable texts and software, and a leader in the mathematical community.

Abstract: For A a positive definite, symmetric n x n matrix and b a real n-vector, the objective function $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^t A\mathbf{x} + \mathbf{b}^t\mathbf{x}$ is optimized over the unit sphere. The proposed iterative methods, based on the gradient of f, converge in general for maximization and for large |b| for minimization with the principal computational cost being one or two matrix-vector multiplications per iteration. The rate of convergence improves as |b| increases, becoming computationally competitive in that case with algorithms developed for the more general problem wherein A may be indefinite.

Keywords: constrained optimization, quadratic functions, iterated gradients, acceleration of convergence.