

## Spring-block models and highway traffic

Z. Nédá<sup>(1)</sup>, F. Járαι-Szabó<sup>(1)</sup>, E. Káptalan<sup>(1)</sup> and R. Mahnke<sup>(2)</sup>

<sup>(1)</sup> Babeş-Bolyai University, Dept. of Physics, RO-400084, Cluj-Napoca, Romania (e-mail: zneda@phys.ubbcluj.ro)

<sup>(2)</sup> Rostock, University, Inst. of Physics, D-10851, Rostock, Germany (e-mail: reinhard.mahnke@uni-rostock.de)

**Abstract:** A brief review on the applicability of spring-block type models for describing various complex systems is given. Pattern formation in drying granular materials, capillarity driven self-organization of nano-particles and magnetization phenomena of ferromagnetic materials are examples in such sense. We learn from these that the spring-block stick-slip model family is especially useful for those phenomena where avalanche-like jumps or self-organized criticality is present. As a novel application, in the present study, a simple one-dimensional spring-block chain with asymmetric interactions is used to model the motion of a queue of cars. Within the model the spring-block chain is dragged with constant velocity through the first block. As a result of this, the blocks in the chain will move in avalanches of widely different magnitudes. For a given parameter range and certain blocks in the system self-organized criticality and disorder-driven dynamical phase transitions are observed.

**Keywords:** complexity, fractal structures, spring-block model, self-organization, highway traffic model

### 1. INTRODUCTION

A simple model of physics with a wide applicability in natural sciences is the spring-block or stick-slip type model. The model was first used by R. Burridge and L. Knopoff in 1967 (Burridge et al., 1967) to explain the empirical law of Gutenberg and Richter (Guttenberg et al., 1956) for the size distribution of the earthquakes. The model consists of simple units: blocks interconnected by springs which are allowed to slide with friction on a plane. The sliding tectonic plates involved in the earthquake were modeled by two surfaces such that their relative movement is governed through the avalanche-like motion of the interconnected blocks (Figure 1). The upper plane (to which the blocks are connected by springs) is dragged with a constant velocity. As a result of this the blocks will slide in avalanches following the motion of the upper surface. The avalanches generated through the sliding blocks model the earthquakes. The energy dissipation through the avalanches exhibits a power-law distribution. This is in good agreement with the empirical law of Gutenberg and Richter.

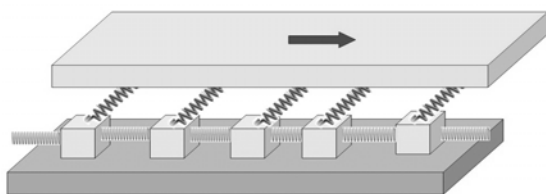


Fig. 1. Main elements of the one-dimensional Burridge-Knopoff model.

The model was generalized in two dimensions by Olami, Feder and Christensen (Olami et al., 1992). Afterwards, due to the spectacular evolution of computers and computer

simulation methods, the spring-block model proved to be useful in describing various other physical phenomena as well. Nowadays, it is a well established fact that the model is extremely useful for studying various mesoscopic phenomena in condensed matter physics. The model is especially suited for those situations where avalanche-like dynamics or pattern formation is present. Known examples in this respect are the PLC (Portevin-Le Chatelier) phenomena (Lebyodkin et al., 1995), fragmentation and fracture of various materials under different conditions (Andersen et al., 1994a, 1994b), or structures formed by the capillary self-organization of nano-particle systems (Chen et al., 2005). Here, we first review some of our recent applications of the spring-block system. After this we present a novel application in modelling highway traffic.

### 2. A BRIEF REVIEW OF RECENT APPLICATIONS OF THE SPRING-BLOCK MODEL.

#### a. Pattern formation in drying granular materials.

Fascinating polygonal patterns obtained in dried granular materials (dried mud for example) are familiar to everyone (Figure 2, bottom row). It is less known however, that such patterns hide an interesting scaling law, which connects the average fragment area with the layer thickness. One successful application of the spring-block type models is the elegant description of these patterns. In this approach the grains of the material are modeled by blocks sliding on the considered two-dimensional substrate, while the capillarity effect of the fluid is modeled by springs interconnecting the blocks (Leung et al., 2000). Initially, the system is subjected to a stochastic stress and relaxation dynamics is imposed on the system. During this dynamics: (i) each block will slide to a new equilibrium position when the total force acting on it is greater than the friction force and (ii) each spring is allowed to break whenever the tension in it exceeds a breaking

threshold. Several layers of springs are considered in order to incorporate the thickness of the material in the model. Due to the competing effects of the spring tensions and frictional forces, blocks will slide in avalanches leading finally to the breakage of the springs and thus to fragmentation of the system. An important scaling relates the average area of fragments with the thickness of the granular layer: the average fragment area scales with the square of the layer thickness. Simple experiments and results of the spring-block model both confirm this general law (Leung et al., 2000). By using the spring-block model one could gain also precious information about the role of the main controllable physical parameters in the final pattern structure. The model reproduced nicely the dependence of the fragment structure as a function of the layer's thickness (Figure. 2, upper row).

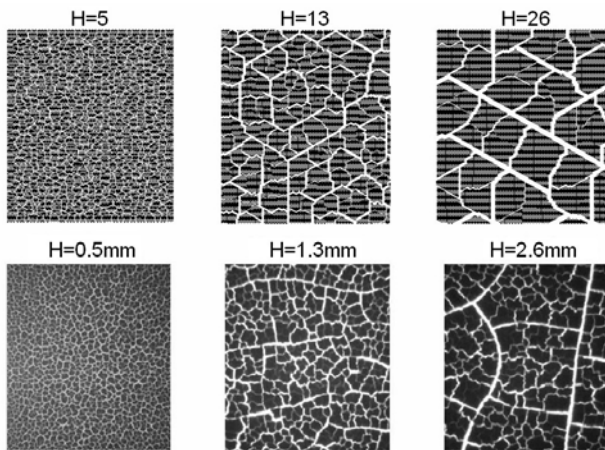


Fig. 2. Visual comparison of real structures obtained through fragmentation of corn-starch (bottom row of pictures) and structures obtained by simulation of the spring-block type model (upper row of pictures) for proportionally selected layer thickness.

By using a spring-block model similar with the one used for the fragmentation of granular materials, and introducing an additional drying front (stress front) moving towards the centre of the two-dimensional system, the formation of curious spiral and other continuously bending fracture structures could be also explained (Leung et al., 2001, Neda et al., 2002). These highly non-trivial structures were experimentally observed by our group when studying the fragmentation of wet precipitates (Figure 3).

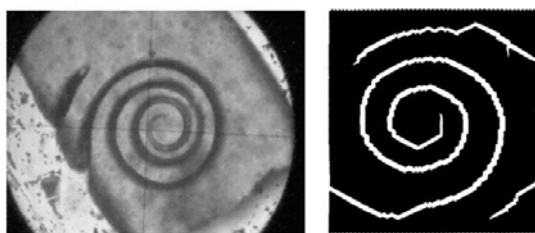


Fig. 3. Spiral cracks detected experimentally in drying precipitates (left) and reproduced by simulations with a simple spring-block type model (right).

### b. Capillarity driven self-assembly of nanostructures

Self-organization is a timely and widely applied method for engineering structures on nanoscale. Self organization can be achieved by several methods, one among them is to use the capillarity forces that appear during the evaporation of the liquid layer. The self-organized structures can be than chemically treated or thin metallic films can be deposited on them, such that the final nanostructure becomes stable and suitable for various practical applications. Nanosphere Lithography is one of such methods. Here polystyrene nanospheres are used as elementary building blocks. In order to obtain various structures with better and better properties, it is important however to further control the capillarity-driven self-organization process of the nanospheres. It is also crucial to understand the mechanisms that can modify or influence the final structures. Beside these, new methods are needed to be developed for reducing the amount of defects in the obtained structures. Our recent studies (Jarai-Szabo et. al., 2005, 2006, 2007) proved that the spring-block type model can be easily modified such that this practically important pattern formation processes are described in a realistic manner. In this approach the nanospheres are modeled with blocks, the capillarity forces between the spheres are modeled by classical springs, and the pinning with the substrate is modeled through static friction forces. Due to the fact that the nanospheres carry electric charges, an additional repulsive force is introduced in the model, which prevents the sliding of blocks on each other. In order to reproduce different crystallization symmetries observed experimentally (triangular and square lattices), we did not use a predefined lattice structure for interconnecting the blocks. The springs connect the blocks according to a random lattice defined by the initial random position of the nanospheres. The spring-block system used in this case follows again relaxation dynamics, similar with the one used in the simulation of drying granular materials. The main difference is, that due to the complex topology of the springs, one cannot implement the problem on a simple cellular automaton (as it was done for the fragmentation case) and molecular dynamics simulations are needed. Simulations are much slower in such case and huge computational power is needed. Simulation results reproduced nicely the experimentally observed structures. The excellent agreement between reality and simulations is illustrated on Figure. 4.

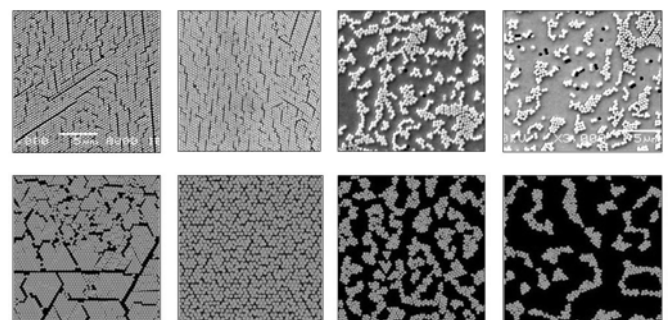


Fig. 4. Capillarity-driven self-assembled nanosphere structures. The upper row shows experimentally obtained structures for different nanosphere densities, lower row

shows simulation results obtained with the spring-block model.

Beyond the excellent reproduction of the experimentally obtained nanosphere structures we have also studied the influence of the relevant and experimentally controllable parameters. It was shown (Jarai-Szabo et al., 2007) that the degree of order in the final structure can be significantly increased and many defects can be eliminated by introducing an additional random force. Experimental results where the system was sonicated during drying, have confirmed the feasibility of this method (Jarai-Szabo et al., 2007). Based on the results of the simple spring-block type model an elegant method for the fabrication of nanostructures with a high degree of ordering was suggested.

### c. Magnetization phenomena and Barkhausen noise

The simple spring-block type model is appropriate also for modeling complex nonlinear magnetization phenomena and explaining the statistical properties of Barkhausen noise. A one-dimensional version of the model was implemented, where blocks represented the Bloch-walls separating oppositely oriented magnetic domains ( $\pm$  in our case) and springs modeled the demagnetizing energy of magnetic domains (Kovacs et al., 2005) (see Figure. 5). As it is proved in the work of Kovacs et al. in 2005, this model describes magnetic systems on a mesoscopic scale. All the relevant interactions that govern the dynamics of Bloch-walls are taken into account: exchange interaction responsible for spontaneous magnetic ordering, interaction between the external magnetic field and magnetic momentums, demagnetization, and pinning forces that block the free movement of the walls (it appears due to defects and impurities in the material). The interaction between the external magnetic field and magnetic momentums are taken into account through constant forces that act on the blocks and have opposite orientation for walls separating  $\pm$  and  $\mp$  domains. The demagnetizing energy is modeled by the energy in the springs and it is proportional with the length of each domain. Finally, pinning forces that stops the free movement of domain walls are modeled by randomly distributed pinning centers with random pinning forces following a normal distribution. Walls can occupy only these pinning centers. The magnetization of the system is the algebraic sum of the lengths of  $+$  and  $-$  oriented domains. The system built up in such manner follows a relaxation dynamics again.

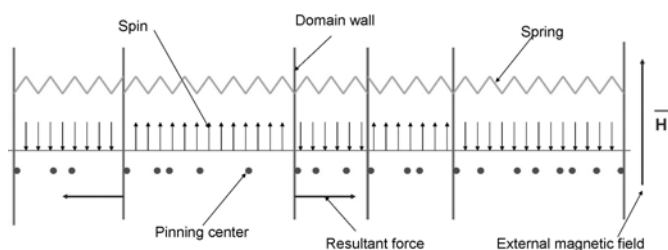


Fig. 5. Main constituents of the spring-block model aimed to reproduce magnetization phenomena of ferromagnets.

Increasing and decreasing the external magnetic field corresponds to changing the force acting on the walls. Whenever the resultant of the external and elastic forces acting on a domain wall exceeds the pinning force, the wall jumps to the next available pinning center which is in the direction of the resultant force. By driving the system through many consecutive magnetization-demagnetization cycles realistic hysteresis loops were obtained (Figure 6a). On the hysteresis loops one can detect many discrete jumps which correspond to the discontinuous variation of the sample's magnetization and generate the so-called Barkhausen noise. As it was shown in the work done by Kovacs and Neda in 2007 the statistics obtained for the Barkhausen noise is realistic.

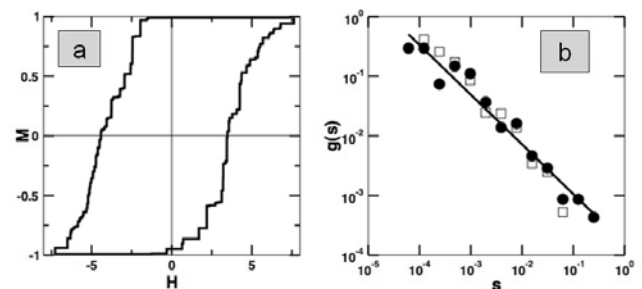


Fig. 6. Hysteresis loop (a) and critical magnetization jump statistics (b) obtained in the simple one-dimensional spring-block model for magnetization

Using this simple one dimensional model for the magnetization processes a special athermal (fluctuationless) phase transition was found. In this phase transition critical behavior appears at a certain critical amount of disorder in the system, and the critical behavior is manifested by power-law distributions of the relevant physical quantities (avalanche sizes for example). Due to the fact that the transition appears as a function of the disorder amount, this phase transition was named as *disorder-driven phase transition* (Kovacs et al., 2007)..

### 3. HIGHWAY TRAFFIC AS A SPRING-BLOCK SYSTEM

Traffic of cars is a truly complex phenomenon where several different forms of collective behavior are detectable (for a review see Kerner, 2004 and Mahnke et al, 2005). One of the simplest forms of traffic is on the highway. An extra simplifying assumption for this problem is when one considers a queue of cars on only one traffic lane, and forbids the cars to leave this lane. The motion of the queue is in such case primarily governed by the leading car. This ideal and simple situation becomes however quite complex if the leading car is moving slowly and the differences between the driving attitudes of the drivers are substantial. In such situation the queue will evolve non-continuously, and jams of various sizes will continuously appear and propagate. Unfortunately such situations are quite common in our everyday life, so understanding it and modeling it is important. Within this study we plan to tackle the problem by considering a spring-block chain model with asymmetric interactions. To our knowledge real experimental data

describing the dynamics of cars in such idealized situation is not available. This makes modeling more difficult, since there is no feedback from real world which would decide if a model is good or bad. This situation is completely different from all previous spring-block modeling attempts described in section 2, where ultimately the predictions of the model were always compared with the experiments. Nevertheless, if a realistic model is developed, the model could offer valuable hints, for what one should measure and look for in the real phenomena. Targeted experiments can be then planned, and both the assumptions of the model and the predicted collective behavior can be verified. The present work intends to motivate such experimental works.

The spring-block approach to this idealized highway traffic problem is straightforward. A spring-block chain is considered (Figure 7), where the blocks model the cars in the row, and the spring models distance keeping interactions between the cars. This distance keeping interaction is the desire of the drivers to keep an optimal distance from the car ahead. It is not desirable that this distance becomes too small, and also it is not optimal to let a too big distance. Of, course, the springs cannot be realistic mechanical springs, since the spring-tension has a unidirectional effect: solely on the car in the back. It cannot have any action on the car in the front, since this is never pulled back or pushed ahead by the car from behind unless there is a collision. In this sense our spring-block system is not a typical mechanical system, where the action-reaction principle is violated.

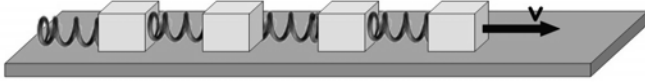


Fig. 7 The spring-block chain considered for modeling idealized highway traffic on one lane.

The crucial effect which will generate disorder in the dynamics of the cars in the row is the inertia of the drivers. This is introduced via friction forces between the sliding blocks and the plane beneath. Friction forces are in fact key ingredients of any spring-block model. In analogy with real mechanical systems we consider here two different types of friction forces: static and dynamic ones. The static friction force is denoted by  $F_s$  and the dynamical one is denoted by  $F_d$ . As usual,  $F_s > F_d$ , and here, their ratio is taken constant:  $F_d / F_s = f$ . We have chosen for all our simulations  $f=0.6$ . Of, course this  $f$  parameter is also an important free parameter of the model, and its influence on the dynamics of the system should be thoroughly investigated. This aspect is planned however for a future study and in the present work we limit our study for a fixed  $f$  value. Apart of distance keeping interactions and inertia of the drivers the disorder (difference between drivers and cars) is also a key ingredient which has to be taken into account within the model. Disorder is of two different types: spatial and time-like. Spatial disorder means differences in the driver personalities, and time-like disorder means the fluctuations of

the driving attitude in time. These disorders can be introduced in several ways. It can be introduced either in the spring constants, or in the friction forces. We have chosen the lately, and treated the spatial and time-like disorder in a unified manner. At each new location (coordinate) of the car (block) we have generated randomly a static friction force value from a normal distribution with a fixed mean ( $F_m$ ) and standard deviation ( $\sigma$ ). Of course, only non-negative  $F_s$  values are accepted. Accordingly, the dynamic friction value of the car is updated too. This means that both in time and space the dynamic and static friction value of the cars will fluctuate.

#### 4. PARAMETERS AND DYNAMICS OF THE MODEL

The force unit in the model has been chosen by considering all springs with spring-constant equal one:  $k = 1$ . The length unit is imposed by the equilibrium length of the springs, which are chosen as:  $l_0 = 1$ . With the force unit imposed by the value of  $k$ , we fix  $F_m = 2$ , and consider  $\sigma$  as the main parameter governing the disorder in the system. Simulations are done in discrete steps. Each step corresponds to a unit time, fixing the time unit. The drag speed (the velocity of the first car) is constant in time. The first block moves ahead in steps of length  $d_0$ . This step length defines the  $v = d_0$  drag velocity. The blocks are labeled after their ordinal number, so that the dragged block has label: 1, the next one label 2, and so on. Step of the block “j” in the row at time moment  $t$ , depends on the distance from the block “j-1” ahead of it ( $x_{j-1}(t) - x_j(t)$ ), and its previous step  $d_j(t-1)$ .

form the chain with distance  $l_0$  between the neighboring blocks:

$$x_j(0) = -j \cdot l_0; \quad d_j(0) = 0$$

move the first block with step  $d_1(t) = d_0$ ,  
 $x_1(t) = x_1(t-1) + d_1(t)$ .

go over each block in the chain, after their ordinal number (label),  $j$

Calculate the spring force acting on block “j”:  
 $T_j(t) = (x_{j-1}(t-1) - x_j(t)) - l_0$

**If block “j” has been moving in the previous step**  
 $d_j(t-1) \neq 0$  generate randomly a new friction force

$F_{sj}(t)$  and correspondingly:  $F_{dj}(t) = f \cdot F_{sj}(t)$ .

Updating the position of the block is done as:

$$x_j(t) = x_j(t-1) + d_j(t-1) + T_j(t) - F_{dj}(t)$$

$$d_j(t) = x_j(t) - x_j(t-1)$$

(this equation corresponds to the laws of classical dynamics with  $\Delta t = 1$ ).

The above update of the coordinates,  $x_j(t)$ , should respect however the following restrictions:

The blocks (cars) can move only ahead:  $d_j(t) = x_j(t) - x_j(t-1) > 0$ , otherwise we leave the coordinate unchanged:  $(x_j(t) = x_j(t-1) \text{ and } d_j(t) = 0)$

The distance between two blocks cannot be smaller than a  $d_{\min}$  value, so if:  $x_{j-1}(t) - x_j(t) < d_{\min} \Rightarrow \Rightarrow x_j(t) = x_{j-1}(t) - d_{\min}$ , and  $d_j(t) = d_{\min}$ . This condition imposes the natural trend of drivers to keep a minimum distance from the car ahead. In our simulations we considered  $d_{\min} = 0.3$

Finally, there is a speed limit for the cars. The movement of the blocks at each time step has a maximal limiting value:

$x_j(t) - x_j(t-1) > d_{\max} \Rightarrow d_{\max}$ . If  $\Rightarrow x_j(t) = x_j(t-1) + d_{\max}$  and

$d_j(t) = d_{\max}$ . This maximal speed was chosen in our simulations as  $d_{\max} = 1$ .

If  $d_j(t-1) = 0$  (the block was not moving), then consider:  $F_{sj}(t) = F_{sj}(t-1)$ . We then calculate the total force  $F_j(t) = T_j(t) - F_{sj}(t)$  acting on the block. If  $F_j(t) > 0$  we use the same equations for the dynamics of block "j" as in item 5 with  $d_j(t-1) = 0$ . Otherwise ( $F_j(t) \leq 0$ ) the block remains in its previous position:  $x_j(t) = x_j(t-1)$  and  $d_j(t) = 0$ .

Update the position of each block,  $j \in \{1, 2, \dots, N\}$ , respecting the rules formulated in items 4-7. Collect the relevant data for the dynamics of the system. These relevant parameters will be described in the next section of the study.

Update the time:  $t \rightarrow t+1$  and repeat the dynamics from item 2.

The above dynamics (1-8.) can be easily implemented as a graphical simulation, so that the dynamics is easy to follow visually. A simple JAVA applet for such a simulation can be downloaded (Néda and Járá-Szabó, 2009). Some typical car positions relative to the first car are illustrated in Figure 8.

The model as described above will have three free parameters:  $d_0$ , the speed of the first car, the  $\sigma$  disorder level in the static friction (reaction time) and the length of the car chain,  $N$ . All the other free model parameters have been fixed as:  $F_m = 2$ ,  $d_{\min} = 0.3$  and  $d_{\max} = 1$ .

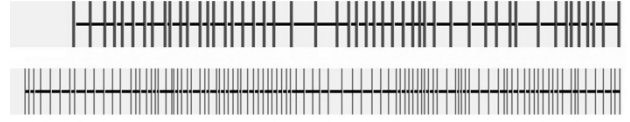


Fig. 8. Snapshot of the chain dynamics. Vertical bars on the horizontal line denote the position of the blocks. The top image is for a chain with  $N=50$  blocks and the bottom one is for  $N=100$  blocks.

## 5. QUANTITIES OF INTEREST FOR THE DYNAMICS OF THE ENSEMBLE

Quantities relevant from the viewpoint of one single car and quantities relevant for the whole chain will be both of interest to us. The quantities we will focus on are the ones that could be measured experimentally and could have practical implications as well.

From the viewpoint of one car in the chain we are interested in its *stop-time distribution*. This stop-time distribution  $g_j(s)$ , describes the distribution of time intervals,  $s$ , the car with ordinal number "j" is not moving.. Since the time step is discrete in our simulation ( $\Delta t = 1$ ), the time-distribution is defined only for discrete, integer variables  $s \in \{1, 2, \dots\}$ . In other words, the  $g(s)$  function gives us the probability that the rest time of the car is  $s$ . The normalized stop-time distribution has the property:

$\sum_{s=1}^{\infty} g(s) = 1$ . Alternatively one can define the cumulative distribution function  $P_{>}(s)$  which gives us the probability that the rest-time is bigger than  $s$ . Naturally:  $P_{>}(0) = 1$ .

A quantity characterizing at a given time moment the whole chain is the *distribution of the jam-size*  $q(k)$ . We define the jam-size,  $k$ , as the number of consecutive cars in the chain that are not moving. Since the number of cars is also a discrete variable, we deal again with a discrete distribution. For each time-moment one can define such a distribution. Assuming that the chain reaches a steady-state dynamics, this distribution should become invariant in time. In order to get a good statistics even for reasonable length chains of cars we consider thus a time average over several snapshots. Normalization of this distribution function is again

straightforward:  $\sum_{k=1}^N q(k) = 1$ . A time-averaged cumulative

distribution of the jam sizes  $P_{>}(k)$  can be also defined. This will give us the probability to have jams of sizes bigger than  $k$ . Naturally:  $P_{>}(0) = 1$ .

Finally, another global quantity of interest is the end-to-end chain length. As time evolves this end-to-end distance,  $h(t) = x_1(t) - x_N(t)$ , will change, and one can define a distribution that characterizes the fluctuation of  $h(t)$ . Since

$h$  is a continuous variable, we can define now a probability density,  $w(h)$ , which gives us the probability that the end-to-end distance is between  $h$  and  $h + \Delta h$ , for a unit  $\Delta h$  value:  $w(h) = P(h, h + \Delta h) / \Delta h$ .

Results for the above defined distribution function are presented and discussed in the next section.

## 6. RESULTS OF THE SPRING-BLOCK MODEL FOR HIGHWAY TRAFFIC

The relevant distribution functions defined in the previous section were thoroughly investigated using a user-friendly and interactive JAVA code (Néda and Járαι-Szabó, 2009). This code plots graphically the position of the cars in the row as illustrated in Figure 8 and determined the evolution of all the relevant distribution functions. Results obtained for various model parameters and system sizes are discussed below.

Let us first investigate the  $P_>(s)$  cumulative stop-time distribution for various cars in a row fixing the drag velocity  $v = d_0 = 0.3$  and the disorder in the static friction:  $\sigma = 2$ . The stop-time distribution varies as a function of the car position,  $n$ , as illustrated in Figure 9.

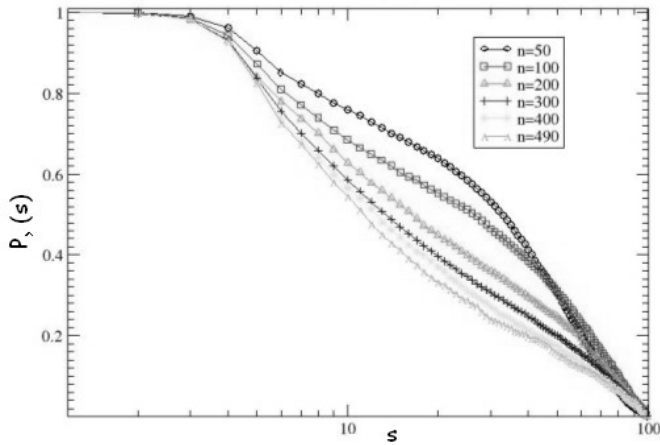


Fig. 9 Cumulative stop-time distribution functions for various cars,  $n$ , in the row on normal-log scale ( $v = d_0 = 0.3$  and  $\sigma = 2$ ).

From these results we learn that for a given car the cumulative distribution function becomes a logarithmic one, suggesting a  $C/x$  power-law type  $g(s)$  distribution function in the limit of large  $s$  values. Since this distribution cannot be normalized in the limit of  $s \rightarrow 0$  or  $s \rightarrow \infty$  it will lead to  $C \rightarrow 0$  for an infinite long dynamics, suggesting a kind of critical states where we all stop-times are significantly possible.

One can look on this distribution from another perspective, fixing the position of the car in the row ( $n = 300$ ) and studying the influence of the drag speed  $v$ . Results in this

sense are presented on both log-normal and normal-log scales on Figure 10.

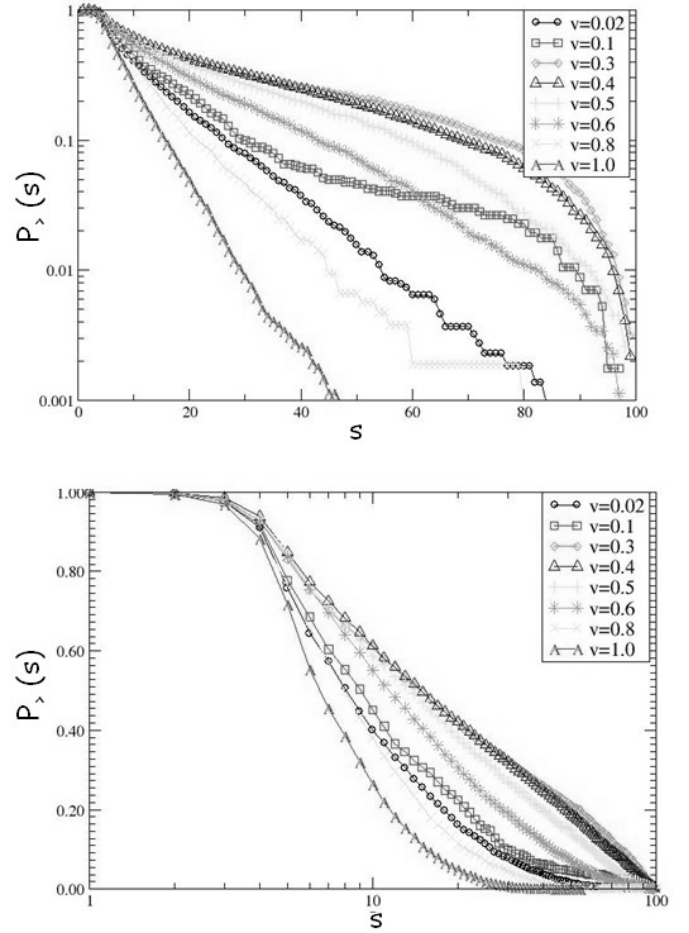


Fig.10. Cumulative stop-time distribution functions for car  $n = 300$  in the row. Results for various drag velocities,  $v$ . The top figure plots this distribution on log-normal scale, the bottom figure plots it on normal-log scale.

The plots in Figure 10 suggest a result similar with our previous conclusion. We find now that for a given car in the row there is a critical (worst) drag velocity for which the distribution function will become a  $C/x$  type power-law. Figure 10 shows that for very small or high drag velocities the cumulative distribution function is exponential. There is thus a critical drag velocity for a car in the row, where the stop-times are largely fluctuating. The given car is in a critical state, and one can consider this phenomenon as a kind of self-organized criticality.

It is also instructive to study the stop-time distribution for a given car in the row as a function of the disorder level quantified by  $\sigma$ . We consider thus the same car in the row ( $n = 300$ ), fix the value of  $v = d_0 = 0.3$ , and consider several different disorder levels in the system. Simulation results are similar with the ones presented in Figures 9 and 10. Results for three different disorder levels are plotted on Figure 11.

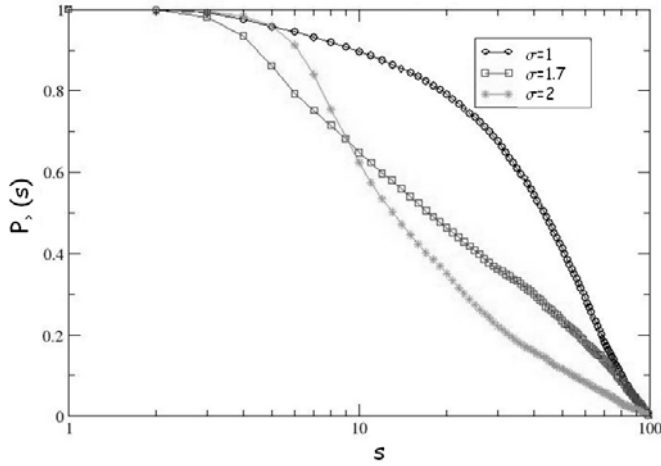


Fig. 11. Cumulative stop-time distribution functions for car  $n = 300$  in the row. Results for various disorder levels:  $\sigma$ . Note the horizontal logarithmic axes.

From Figure 11 it results that for a given disorder level, the car will have a cumulative stop-time distribution of logarithmic nature ( $\sigma = 1.7$ ). This means  $g(s) \approx C/X$ , and the stop-time will exhibit a fat-tail power-law distribution. Since this critical state appears for a given disorder level, we can consider it as a disorder-induced phase transition, similar with the one obtained in our spring-block model for magnetization (Kovács and Nédá, 2007). Studying thus the stop-time distributions for a given car in the row as a function of the velocity of the first car and the disorder in the system, we found two interesting collective behaviors: self-organized criticality and a disorder induced critical state.

The jam-size distribution follows a different trend. Contrary to the stop-time distribution this is a distribution that characterizes the whole chain and not only one car. Fixing the value of disorder ( $\sigma = 2$ ) we studied the influence of the  $v$  drag velocity. Results for the cumulative distribution function are plotted on Figure 12.

Results plotted on Figure 12 suggest that the jam-size distribution is exponential if the drag velocity approaches the  $v = d_0 = 1$  limiting value. It is easy to verify that for smaller velocities there is no power-law or logarithmic trend. The shape of the  $P_>(j)$  curves shows a monotonic trend towards this exponential form. Seemingly the jam-size distribution does not give any hint for a critical behavior of the system as a whole.

Very similar results are obtained for the end-to-end distance's statistics of the chain. The  $w(h)$  probability density function will exhibit an exponential form in the low drag velocity limit (Figure 13). There are however no drag velocities or disorder levels which would ensure a power-law type distribution function.

Simulation results revealed thus interesting nontrivial behavior for the dynamics of one car in the row. No hints for critical collective behavior of the chain as a whole were obtained up to now. The stop-time distribution of cars seems

to be thus the relevant information, which becomes critical for a given position in the row, drag velocity and disorder amount.

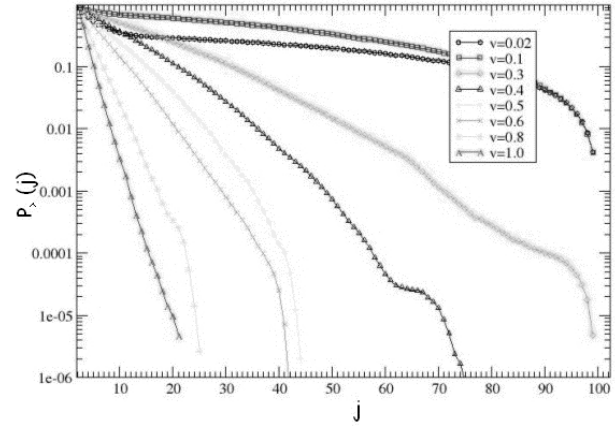


Fig. 12 Cumulative distribution functions of the jam-size values, for various drag velocities,  $v$ . Please note the log-normal scales.

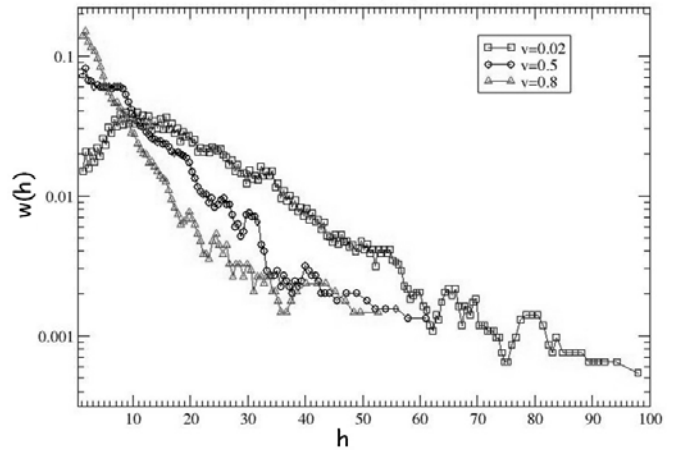


Fig. 13 Probability density of the end-to-end distance distribution for various drag velocities of the chain. Please note the log-normal scale.

## 7. CONCLUSIONS

The primarily aim of the present paper was to attract the attention of the scientific community on the vast and interdisciplinary applicability of the simple spring-block type model. As it is illustrated through the discussed examples the spring-block system is nowadays successfully used for modeling complex phenomena in nature. It is particularly useful for those phenomena where avalanche-like processes or self-organized critically is expected. As a novel application, it was used here for modeling idealized highway traffic. The spring-block chain dragged with a constant speed, predicts interesting and rich dynamics for single lane highway traffic. For low velocities of the leading car and high enough disorder level in the driver attitudes, the cars will evolve non-continuously, alternating advancing and halting regimes. For each car in the row there is a worst

velocity of the leading car when it's stop-time distribution becomes fat-tail. Alternatively, for a fixed velocity of the leading car there is a critical disorder level in the driver attitudes, for which the stop-time distribution of the considered car will become critical. Signatures of self-organized criticality and disorder-driven criticality are thus predicted for this idealized system. Experiments which would confirm or infirm the prediction of this model should be considered in the future.

#### ACKNOWLEDGMENT

Work supported from a PN2/IDEI nr. 2369 Romanian research grant.

#### REFERENCES

- Andersen J. V., Brechet Y. and Jensen H. J. (1994). Fracturing described by a spring-block model, *Europhys. Lett.*, **26**, 13-18.
- Andersen J. V. (1994). Dynamical Mean Field theory for a spring-block model of fracture, *Phys. Rev. B*, **49**, 9981-9984.
- Baik J.M. and Thompson R.B. (1984). Ultrasonic scattering from imperfect interfaces, *J. Nondestr. Eval.*, **4**, 177-196
- Burridge R. and Knopoff L.(1967). Model and theoretical seismicity, *Bull. Seism. Soc. Am*, **57**, 341-371.
- Chen K., Taflove A., Kim Y.L. and Backman V. (2005). Self-assembled patterns of nanospheres with symmetries from submicrons to centimeters, *Appl. Physics Letters*, **86**, Art. Nr. 033101.
- Delsanto P.P. and Scalerandi M. (1998) A spring Model for the simulation of wave propagation across imperfect interfaces., *J. Acoust. Soc. Am.*, **104**, 2584-2591
- Gutenberg B. and Richter C.F.(1956). Magnitude and energy of Earthquakes., *Ann. Geofis.*, **9**, 1-23 .
- Járai-Szabó F., Astilean S. and Nédá Z. (2005). Understanding self-assembled nanosphere patterns, *Chem. Phys. Lett.*, **408**, 241-246.
- Járai-Szabó F., Kuttesch A., Astilean S., Nédá Z., Chakrapani N., Ajayan P.M. and Vajtai R. (2006). Spring-block models for capillarity driven self-organized nanostructures. *J. Opt. Adv. Mat.*, **8**, 1083-1087.
- Járai-Szabó F., Nédá Z., Astilean S. , Farcau C. and Kuttesch A. (2007). Shake-induced order in nanosphere systems. *Eur. Phys. J. E*, **23**, 153-159.
- Kerner B.S. (2004). *The physics of Traffic* (Springer, Berlin, New York).
- Kovács K., Brechet Y. and Nédá Z (2005). A spring-block model for Barkhausen noise, *Mod. Sim. Mat. Sci. Eng.* , **13**, 1341-1352 .
- Kovács K. and Nédá Z. (2007). Disorder-driven phase transition in a spring-block type magnetization model, *Phys. Lett. A*, **361**, 18-23.
- Lebyodkin M.A., Brechet Y., Estrin Y., and Kubin L.P.(1995). Statistics of the catastrophic slip events in the Portevin-Le Chatelier effect, *Phys. Rev. Lett.* **74**, 4758 – 4761.
- Leung K.T. and Nédá Z. (2000). Pattern formation and selection in quasi-static fracture, *Phys. Rev. Lett.*, **85**, 662-665.
- Leung K.T, Andersen J.V and Sornette D. (1998), Self-organized criticality in stick-slip models with periodic boundary conditions, *Phys. Rev. Lett.*, **80**, 1916-1919
- Leung, K.T., Józsa L., Ravasz M. and Nédá Z.,(2001). Spiral cracks without twisting, *Nature*, **410**, 166-166.
- Lu Y.N. and Ding E.J., (1993). Self-organized criticality in a stochastic spring-block model. *Phys. Rev. E*, **48**, R21-R24
- Mahnke R., Kaupuzs J., Lubashevsky I. (2005). Probabilistic description of traffic flow, *Phys. Rep.*, **408**, 1-130
- Nédá Z., Leung K.T., Józsa L. and Ravasz M.(2002). Spiral cracks in drying precipitates, *Phys. Rev. Lett.*, **88**, Art. No. 095502
- Nédá Z. and Járai-Szabó F. .(2009). Spring-block model for traffic flow: <http://atom.ubbcluj.ro/traffic>
- Olami Z., Feder J.S. and Christensen K. (1992). Self-organized criticality in a continuous, nonconservative cellular automaton modeling earthquakes, *Phys. Rev. Lett.*, **68**, 1244-1247